

To the memory of G.T. Zatsepin

Neutrino oscillations: Entanglement, energy-momentum conservation and QFT

E. Kh. Akhmedov^{a,b*} and A. Yu. Smirnov^{c†}

^a*Max-Planck-Institut für Kernphysik, Postfach 103980
D-69029 Heidelberg, Germany*

^b*National Research Centre Kurchatov Institute
Moscow, Russia*

^c*The Abdus Salam International Centre for Theoretical Physics
Strada Costiera 11, I-34014 Trieste, Italy*

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Abstract

We consider several subtle aspects of the theory of neutrino oscillations which have been under discussion recently. We show that the S -matrix formalism of quantum field theory can adequately describe neutrino oscillations if correct physics conditions are imposed. This includes space-time localization of the neutrino production and detection processes. Space-time diagrams are introduced, which characterize this localization and illustrate the coherence issues of neutrino oscillations. We discuss two approaches to calculations of the transition amplitudes, which allow different physics interpretations: (i) using configuration-space wave packets for the involved particles, which leads to approximate conservation laws for their mean energies and momenta; (ii) calculating first a plane-wave amplitude of the process, which exhibits exact energy-momentum conservation, and then convoluting it with the momentum-space wave packets of the involved particles. We show that these two approaches are equivalent. Kinematic entanglement (which is invoked to ensure exact energy-momentum conservation in neutrino oscillations) and subsequent disentanglement of the neutrinos and recoiling states are in fact irrelevant when the wave packets are considered. We demonstrate that the contribution of the recoil particle to the oscillation phase is negligible provided that the coherence conditions for neutrino production and detection are satisfied. Unlike in the previous situation, the phases of both neutrinos from Z^0 decay are important, leading to a realization of the Einstein-Podolsky-Rosen paradox.

*email: akhmedov@mpi-hd.mpg.de

†email: smirnov@ictp.it

1 Introduction

Although neutrino oscillations appear to be a simple quantum mechanical phenomenon, a closer look at them reveals a number of subtle and even paradoxical issues. It is probably for this reason that debates on the fundamentals of the oscillation theory and on the correctness of different theoretical approaches to neutrino oscillations do not cease in the literature, with many papers published in the recent years. Some of these publications advocate approaches that in our opinion are either incorrect or confusing.

Unfortunately, almost every new attempt to revise the basics of the theory of neutrino oscillations or to re-interpret the already established results is either incorrect or leads to a more complicated and less transparent than before formalism, often with many unnecessary details. This hinders the understanding of the physics of the oscillation phenomenon and, in turn, triggers further incorrect developments. Some confusion originates from the fact that there exist several seemingly different (though actually equivalent) approaches to neutrino oscillations, which allow different physics interpretations.

In this paper we consider several subtle issues of the theory of neutrino oscillations which have been under discussion recently. These include inter-related questions of the energy-momentum conservation in neutrino oscillations, entanglement of neutrinos and accompanying (“recoil”) particles, and the possibility of describing neutrino oscillations in the S -matrix formalism. We present our analysis in the simplest and most transparent, yet physically adequate way, omitting irrelevant details. In the course of our discussion we also comment on some incorrect considerations and results which recently appeared in the literature. Attempts at implementation in neutrino oscillations of exact energy-momentum conservation, kinematic entanglement, etc., lead to a number of paradoxes, for which we present our resolution. In this sense the present paper can be considered as a continuation of our previous work “Paradoxes of neutrino oscillations” [1].

The paper is organized as follows. In sec. 2 we show how neutrino oscillations can be consistently and adequately described in the S -matrix formalism of quantum field theory (QFT). Sec. 3 is devoted to the problem of energy-momentum conservation in neutrino oscillations in the QFT framework. In sec. 4 we consider kinematic entanglement and disentanglement in neutrino oscillations and show in this connection that the phase of the recoil particle is irrelevant. Here we also discuss relationships between neutrino oscillations and the Einstein-Podolsky-Rosen paradox. In the Appendices we present our critical comments on some recent papers on neutrino oscillations.

2 S -matrix approach of QFT and the physics conditions of neutrino oscillations

Can neutrino oscillations be consistently described in the standard S -matrix formalism of QFT and, if not, how should this formalism be modified to make the description possible? These questions have been addressed in a number of publications. Recently, it has been argued [2, 3] that the S -matrix formalism is ill-suited for describing long-baseline oscillation experiments because the oscillations are inherently a finite-time phenomenon. This is at variance with the results of a number of earlier papers, in which it was shown that in the framework of a QFT approach with wave packets, finite-distance and finite-time *neutrino* evolution can be consistently incorporated into the S -matrix formalism (see [4] for a comprehensive review). Here we present our own analysis of this issue and confirm the latter statement. Although some points in our discussion may appear rather obvious, the fact that their misconception has led to incorrect results shows that they deserve a detailed consideration.

2.1 Physics conditions of neutrino oscillations

In principle, QFT and the S -matrix formalism should be able to describe any physical process in particle physics. However, in each particular case the formalism has to be adjusted to the specific physical situation by making use of, e.g., appropriate initial conditions. Such an adjustment is carried out for the processes of scattering of particles, particle decays, *etc.*. In addition, calculations usually involve certain approximations that are process dependent, *i.e.* are valid for a given situation but may not be justified for the other ones.

Recall that when one employs QFT to describe a scattering process, the initial state is considered to represent a system of free non-interacting particles. In the process of evolution the particles approach each other, enter certain space-time region where they interact, then the products of the interaction move apart and after some time are again considered as free non-interacting particles. In this setup one deals with a *single interaction region* (see fig. 1a). The following approximations are usually made:

- (i) the initial and the final states are considered as asymptotic states (defined at $t \rightarrow \pm\infty$) and are described by plane waves;
- (ii) the integration over the 4-coordinate of each interaction point is performed over the infinite space-time interval.

This results in the proportionality of the transition amplitude to δ -functions expressing exact energy-momentum conservation in each interaction vertex, which substantially simplifies calculations of rates and cross-sections. The localization of the source and detector is irrelevant in this setting and is not considered. (In fact, the infinite-limits integration and the plane-wave description of the asymptotic states means that both the source of the

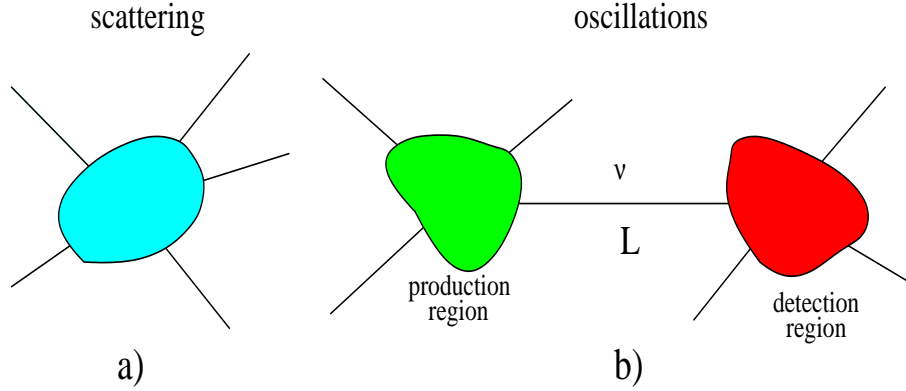


Figure 1: Schematic representation of a scattering process (a), and of a neutrino oscillation setup with distinct production and detection regions separated by a distance L (b).

particles and the detector are formally taken to be of infinite extension).

In contrast to this, neutrino oscillations are intrinsically a finite-time and finite-distance phenomenon. Neutrinos are produced in a certain confined space-time region (the source), then they propagate and are detected in another confined space-time region – the detector. The detector and the source have *finite* sizes and are separated by a *finite* distance L (fig. 1b). Usually this distance is much larger than the sizes of the production and detections regions. The existence in the problem of a finite length – the baseline L – constitutes a fundamental difference from scattering processes. Thus, in the case of neutrino oscillations one deals with two distinct interaction regions. (The situation when the production and the detection regions overlap or coincide should be considered separately.) The S -matrix formalism of QFT should be adjusted correspondingly.

To incorporate the two interaction regions setup in the S -matrix formalism one should employ appropriate wave functions (wave packets) for particles accompanying neutrino production and detection (hereafter we will use the term “external” for such particles).¹ The wave functions of particles which are involved in neutrino production ensure the spatial localization of the neutrino emission region in the source, whereas the wave functions of particles with which neutrinos interact upon propagation should determine the localization of the detection region. The approximation of (infinite-extension) plane waves for asymptotic states is not valid here – in that approximation the neutrino source and detector cannot be localized, which results in oscillations being averaged out.

Notice that in the case of scattering processes there may exist several interaction vertices with 4-coordinates $x_i = (t_i, \mathbf{x}_i)$. The interaction can occur anywhere in the interaction

¹The name comes from the fact that they correspond to external legs of the Feynman diagram describing the overall process of neutrino production, propagation and detection, whereas neutrinos are represented by the internal line in this diagram (see fig. 1b).

region, and one therefore has to perform the integration $\int \prod_i d\mathbf{x}_i dt_i$ over the whole space-time interaction domain for each vertex. This leads to the appearance of cross-diagrams. In contrast with this, in the case of neutrino oscillations, in the lowest order in weak interactions there are only two interaction vertices (one for neutrino production and one for its detection) situated in different interaction regions, and the integrations over the corresponding 4-coordinates $x_1 = (t_1, \mathbf{x}_1)$ and $x_2 = (t_2, \mathbf{x}_2)$ are performed over different space-time intervals. To a good approximation these regions are spatially separated and time ordered. There are no cross diagrams in this case – the probability that, e.g., a particle associated with neutrino production will be born in the detector rather than in the source is exponentially suppressed [2]. Let us illustrate the above by some examples.

(i) Reactor experiments: $\bar{\nu}_e$ are produced in decays of nuclei A and then detected via the inverse beta decay $\bar{\nu}_e + p \rightarrow e + n$. The decaying nuclei are localized in the source (the core of a nuclear reactor), and the target protons are localized in the detector. The wave function of a parent nucleus A is substantially different from zero only in a relatively small region with the size of the order of interatomic distances, *i.e.* much smaller than the size of the reactor core. A similar argument applies to the wave functions of target protons. It is this localization of the neutrino emitter and receiver that determines the positions of the two interaction regions; their sizes are given by the overlap of the wave packets of the particles participating in the neutrino production and detection processes (see below).

(ii) Accelerator experiments: neutrinos are produced in pion decays, $\pi \rightarrow \mu + \nu_\mu$, and the electron neutrinos that appear as a result of the oscillations are detected via the $\nu_e + n \rightarrow p + e$ reaction, where the neutron is inside a nucleus. Here the pions decay in a decay tunnel. They are described by moving wave packets whose localization domain may, in fact, be much smaller than the size of the tunnel.

2.2 Space-time diagrams for oscillations

Let us discuss the space-time picture of neutrino production, propagation and detection in more detail. For the process described in example (ii), the evolution of the system is sketched in fig. 2. The colored bands correspond to space-time localization of the participating particles as described by wave packets. The rectangular regions schematically show the overlap domains of the wave packets at neutrino emission and absorption, *i.e.* the neutrino production and detection regions. The propagating neutrino state is a superposition of two different mass eigenstates, ν_1 and ν_2 , moving with slightly different group velocities (to make this more clearly seen, we have shown the borders of the band corresponding to ν_1 with black dashed lines). If neutrinos propagate very long distances, the bands corresponding to different mass eigenstates would no longer overlap at the detector, and the oscillations would become unobservable. This corresponds to decoherence due to the wave packet separation. In fig. 2 the parent pion in the source and the target nucleon (or nucleus) in the detector are assumed to be (nearly) at rest.

Let us elaborate on the space-time characteristics of the interaction regions. Since the range of the weak interactions responsible for neutrino production and detection is extremely small, the interaction regions are determined by the overlap of wave packets of the involved particles. In turn, the shapes and the sizes of the wave packets depend on the specific conditions of experiment. Notice that the wave packets of the external particles do not have sharp borders, and the same applies to the regions of their overlap – the interaction regions.²

In process (ii) the space-time domain occupied by the pion is determined in the following way. The initial time of this domain (the left hand border of the production rectangle in fig. 2) is given by the time when, e.g., a bunch of protons hits the target, i.e when the pions are produced, or when the number of pions in the source is counted. If the pions decay at rest in vacuum, the final moment of time of the domain is essentially determined by the pion lifetime τ_0 . The temporal length of the domain can be smaller if, e.g., the decay tunnel is shorter than the pion decay length for moving pions. The diagram in fig. 2 corresponds to a situation when the produced muon is not detected and no particular space-time conditions are imposed on the detection of the electron and recoil nucleus in the detector. In this case the production region is determined by the properties of the pion wave packet. The detection region is given by the intersection of the bands representing the space-time propagation of the wave packets of the neutrino and the neutron.

The situation can be different if the muon is detected, and a time window is imposed on its detection that is shorter than the pion lifetime. This narrows down the muon wave packet and consequently the interaction region will be determined by the intersection of the pion and muon bands. In this case the temporal length of the production region, and consequently, of the detection region, will be smaller than the one shown in the fig. 2. Similarly, the time or space window imposed on the electron detection may narrow down the detection region, and therefore reduce the size of the neutrino wave packets, and consequently the production region.

If neutrinos are produced in collision processes, the beginning of the interaction region corresponds to the moment of time when the wave packets describing the colliding particles begin to overlap.

The initial (final) moment of time t_S^i (t_S^f) of the production region can be defined in such a way that for $t < t_S^i$ ($t > t_S^f$) the overlap of the wave packets of particles participating in the neutrino production process can be neglected. Recall that the size of the wave packet of neutrino is determined also by the process of neutrino detection. The initial and final moments of time of the detection region, t_D^i and t_D^f , can be defined similarly.

In fig. 2 we indicate the 4-coordinates of the central points of the neutrino production and detection regions, (T_S, \mathbf{X}_S) and (T_D, \mathbf{X}_D) , respectively. They can be defined as follows. Let T_S be the time when the overlap of the wave packets of particles participating in neutrino

²The bands in fig. 2 should actually have fuzzy peripheries. They are depicted with sharp borders just for simplicity of drawing.

production is maximal; \mathbf{X}_S is then the position of the central point of the overlap region at this time. The time T_D and the coordinate of the center of the detection region \mathbf{X}_D can be defined similarly. The baseline L is then given by the modulus of the vector $\mathbf{L} = \mathbf{X}_D - \mathbf{X}_S$, whereas the mean time elapsed between the neutrino production and detection is $T = T_D - T_S$. These quantities are well defined as long as they are large compared to, correspondingly, the spatial and temporal extensions of the neutrino production and detection regions.

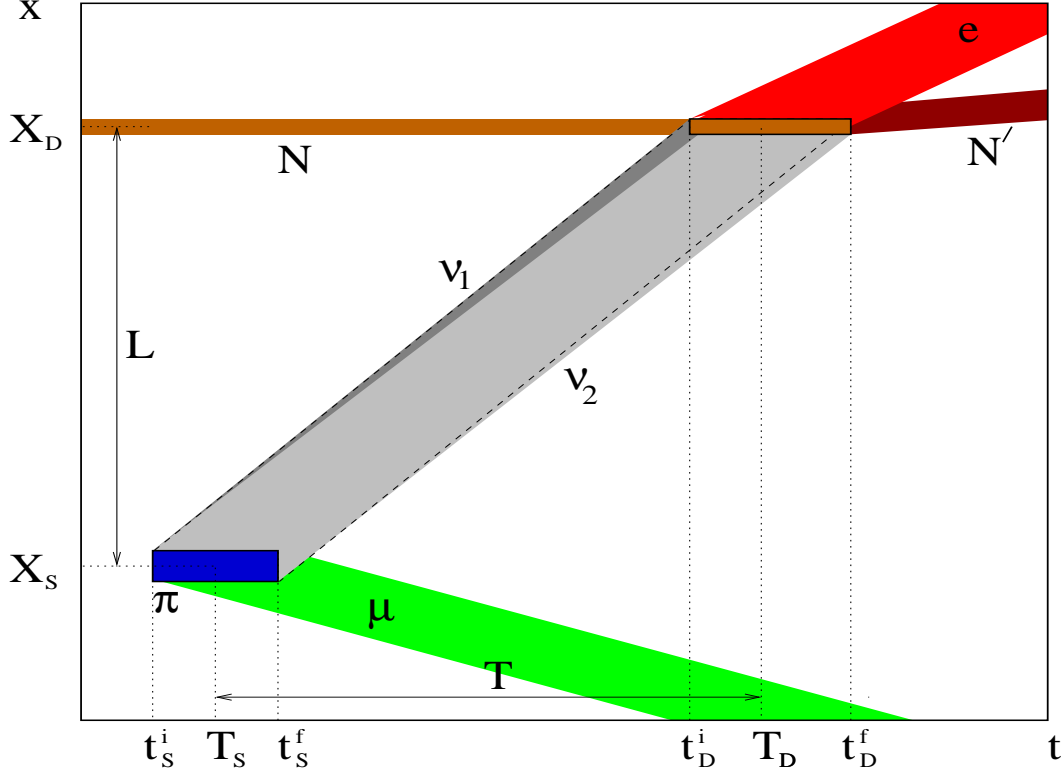


Figure 2: Space-time diagram of neutrino oscillations. Schematic representation of neutrino production in pion decay, propagation and observation of the oscillated neutrino due to its charged-current interaction with a target nucleon (or nucleus). See the text for details.

Within the S -matrix formalism, the calculation of the transition amplitude involves the integrations over 4-coordinates of the neutrino production and detection points x_1 and x_2 which are performed over the infinite space-time interval. The transition amplitude corresponding to the production and detection of the i th neutrino mass eigenstate can be written as

$$\mathcal{A}_i = \int d^4x_1 \int d^4x_2 \prod_j \Psi_j(x_1, \mathbf{X}_S, T_S) \prod_l \Psi_l(x_2, \mathbf{X}_D, T_D) M_i(x_1, x_2). \quad (1)$$

Here j (l) runs over the external particles interacting in the production point x_1 (detection point x_2), $\Psi_j(x_1, \mathbf{X}_S, T_S)$ and $\Psi_l(x_2, \mathbf{X}_D, T_D)$ are the wave functions (wave packets) of the external particles in the case of incoming particles or complex conjugates of the wave functions in the case of the outgoing particles and, as we discussed, (T_S, \mathbf{X}_S) , (T_D, \mathbf{X}_D) determine the space-time location of the interaction regions. The quantity $M_i(x_1, x_2)$ is the remainder of the transition matrix element. The amplitude of the overall process of ν_α production, propagation (including the oscillations) and ν_β detection is then given by the sum

$$\mathcal{A}_{\alpha\beta}^{tot} = \sum_i U_{\alpha i}^* U_{\beta i} \mathcal{A}_i, \quad (2)$$

where U is the leptonic mixing matrix. The relative phases between the amplitudes \mathcal{A}_i with different i determine the oscillation phases.

The dominant contributions to the integrals in (1) come from finite space-time intervals which correspond to the production and detection domains discussed above. In particular, the infinite-time integrations can be replaced by those over finite time intervals: $t_i^S \leq t_1 \leq t_f^S$ and $t_i^D \leq t_2 \leq t_f^D$, or by any time intervals encompassing them – the result will be practically independent of the integration limits provided that they include the interaction regions.

Alternatively, one can still use the plane wave approximation for the wave functions of the external particles. But in this case, in order to ensure proper localization of the neutrino production and detection processes, one should impose “by hand” the finite integration limits for the 4-coordinates x_1 and x_2 , corresponding to the finite space-time domains of the interactions. In eq. (1) finite *effective* integration intervals were selected automatically. The plane-wave description plus imposition of finite integration limits can be considered as an approximation to the localized wave functions description of neutrino oscillations with infinite-limits integrations over x_1 and x_2 . Using such “finite-extension plane waves” is sometimes invoked to argue that the plane wave approach still gives a viable description of neutrino oscillations. It should be remembered, however, that a finite “piece” of a plane wave is no longer a plane wave – it is a wave packet.³ Any consistent description of neutrino oscillations must therefore employ wave packets for the external particles (or at least for some of them). The plane wave approach cannot describe the localization of the neutrino source and detector, without which neutrino oscillations would be unobservable.

Since the neutrino source and detector are of finite size, in realistic situations one should, in fact, both use wave packets and perform finite-interval integrations. However, if the limits imposed by macroscopic sizes of the source and detector are large compared to the (usually microscopic) space-time regions of the *individual* neutrino production and detection processes, the results obtained with finite-interval integrations practically coincide with those corresponding to infinite-limits integration.

³Indeed, a 1-dimensional “piece” of the spatial size $2a$ of a plane wave with momentum p_0 is actually a wave packet with the momentum distribution function $f(p, p_0) = \text{const} \times \sin[(p - p_0)a]/(p - p_0)$ rather than $f(p, p_0) = \delta(p - p_0)$, which would correspond to a true (infinite-length) plane wave.

2.3 Finite-limits time integration and the observation time

In refs. [2, 3] a finite evolution time approach to neutrino oscillations has been developed, and expressions for the oscillation probabilities that differ from the standard one were obtained. The results of [2, 3] follow from the authors’ use of incorrect physics conditions that do not correspond to a realistic setup of neutrino oscillations. In [2, 3] all the particles involved in neutrino production and detection as well as the neutrinos themselves were described by plane waves, and the integration over the detection time t_2 was performed over the interval which extends from the time of observation of the charged lepton accompanying the neutrino emission in the source to the time of observation of the charged lepton born in the neutrino absorption process in the detector. These instants of time approximately coincide with the times when the neutrino production and detection processes end, *i.e.* with t_S^f and t_D^f in our notation. In other words, in the analysis of [2, 3] the detection region was immediately attached to the production region.

While integrating over a very long detection time interval would be perfectly admissible in the wave packet description of neutrino oscillations (the wave packets overlap would automatically select the correct interaction region), this leads to grave consequences in the plane-wave framework adopted in [2, 3]. As we pointed out in secs. 2.1 and 2.2, the plane wave approximation augmented by judiciously selected finite integration intervals could to some extent imitate the wave packet approach. This, in particular, means that the integrations should be performed over the production and detection regions (the rectangles in fig. 2). The approach of [2, 3], in which the integrations were extended from t_S^f to t_D^f , actually amounts to assuming the neutrino detector to be as long as the experimental baseline itself (see Appendix A for a more detailed discussion).

In refs. [2, 3] the authors confused the total *time of experimental observation* t_{exp} (which they call “the total reaction time”) with the neutrino evolution time T , which is the mean interval of time between the neutrino production and detection. Their expressions for the oscillation probabilities exhibit both proportionality to T and an oscillatory behavior in T . The correct calculations give the proportionality of the overall probability of the process as well as of the event numbers to t_{exp} , and an oscillatory behavior in T [5].

In this connection, let us clarify the meaning of the time of the experimental observation t_{exp} and its connection to the time intervals introduced in secs. 2.1 and 2.2. In practice, t_{exp} is the time interval during which measurements are performed, e.g. the time interval between the moments when the detector is turn on and off, or the time during which the accelerator provides a beam of particles that produce neutrinos. Thus, t_{exp} is essentially the time of continuous accumulation of statistics (at neutrino detection) or luminosity (if we speak about the neutrino production). This time can range from a very small fraction of second, if a special time window for observation is imposed (as, for instance, in K2K and MINOS accelerator experiments), to years, when, e.g., an underground detector accumulates neutrino events.

The dependence of the number of events on t_{exp} and possible relations between t_{exp} and

the detection and production time intervals are determined by the nature of the external states involved in the process.

1. In the case of stationary external states which, in turn, produce stationary neutrino states (e.g., neutrino production in decays of very long lived isotopes) the time t_{exp} is directly related to the detection time: $t_{exp} = t_D^f - t_D^i$. It is assumed here that the detection process is also stationary. The time t_{exp} determines the size of the wave packet of, e.g., the electron in the final state of the example shown in fig. 2 (once the wave packet of the neutron is given), and consequently, the temporal extension of the detection region. Therefore, t_{exp} determines also the limits of integration over times t_1 and t_2 in (1). The length of the integration intervals is identified with the time of experimental observation. If the observation time is much larger than the neutrino propagation time, $t_{exp} \gg T$, the quantity t_{exp} determines approximately the total time of the production-propagation-detection process. The number of detected events is then simply proportional to t_{exp} . Mathematically, this proportionality is a consequence of the proportionality of the transition amplitude to the energy-conserving δ -function $\delta(E_f - E_i)$, which appears due to the *stationary situation* [6]. Indeed, for large t_{exp} the probability is proportional to

$$[\delta(E_f - E_i)]^2 \propto t_{exp} \delta(E_f - E_i). \quad (3)$$

2. The situation is different when the external particles are described by relatively short moving wave packets, so that the lengths of the interaction intervals (t_S^i, t_S^f) and (t_D^i, t_D^f) , which are determined by the overlap times of the corresponding external wave packets, are much smaller than t_{exp} and T . The amplitude of the process is not proportional to the energy-conserving δ -function in this case [4]. Therefore, the probability of the *individual* neutrino production-propagation-detection process is not proportional to the time of observation t_{exp} even in the S -matrix formalism, contrary to the claim in [2, 3]. This is because such a proportionality arises only in a stationary situation, whereas the process with production and detection of a single neutrino is highly non-stationary. The proportionality to t_{obs} can appear, however, if one deals with stationary beams of incoming particles or ensembles of emitters rather than with individual processes (see [5], sec. 5.2). In this case the intervals of integration over t_1 and t_2 in (1) are not related to t_{exp} ; in particular, they can be much smaller than t_{exp} .

In any case, the number of events should not oscillate with t_{exp} . It should, however, oscillate with T – the time elapsed between the production and detection of neutrinos – if the T - and L -dependent probability of the process $P_{\alpha\beta}(T, L) = |\mathcal{A}_{\alpha\beta}^{tot}(T, L)|^2$ is considered. Note that since the neutrino production and detection times are usually not measured (or at least not measured accurately enough), the probability $P_{\alpha\beta}(T, L)$ is often integrated over T . However, this integration is not equivalent to the one in [2, 3] and it does not reproduce the results of those papers. Indeed, $P_{\alpha\beta}(T, L)$ is significantly different from zero only if $T \simeq L/v$, where v is the average group velocity of the neutrino state, with possible deviations from the

exact equality given by the length of the neutrino wave packet σ_x .⁴ In the limit $\sigma_x \rightarrow 0$, the probability becomes proportional to $\delta(L - vT)$, so that the integration simply identifies vT with L . For finite σ_x , the integration over T effectively amounts to averaging the probability over the finite sizes of the production and detection regions. In any case, this integration yields the oscillation probability $P_{\alpha\beta}(L)$ which exhibits an oscillatory behavior only in L .

3 Energy-momentum conservation and neutrino oscillations

3.1 Energy-momentum conservation and localization

As has been pointed out by many authors, the assumption of exact energy and momentum conservation, if immediately applied to particles participating in the neutrino production and detection processes, would make neutrino oscillations impossible. The reason for this is twofold. First, exact energy-momentum conservation would allow one to determine the energy E_ν and momentum \mathbf{p}_ν of neutrino from those of the external particles, provided that energy and momenta of these particles are known precisely. Since neutrinos propagate macroscopic distances and are therefore on the mass shell, one can then determine the neutrino mass from the relation $E_\nu^2 = \mathbf{p}_\nu^2 + m^2$. This would mean that the neutrino state is a mass eigenstate rather than a coherent superposition of different mass eigenstates. Note that mass eigenstates do not oscillate in vacuum. Second, exact energy-momentum conservation implies that all the involved particles have sharp energies and momenta, *i.e.* are described by plane waves. This would make localization of the neutrino source and detector impossible, leading to a washout of neutrino oscillations. The former difficulty could to some extent be alleviated through kinematic entanglement [8, 9, 10, 11] (see the next section), but the latter one remains.

As has been stressed in [1], this does not, of course, mean that energy-momentum conservation, which is a fundamental law of nature, is violated: it is satisfied exactly when one applies it to all particles in the system, including those whose interactions with the particles directly involved in the process in question localize the latter in given space-time regions. In other words, energy and momentum uncertainties are not in contradiction with energy-momentum conservation, which is exact for closed systems.

In most situations (e.g., in scattering experiments), the contributions of the localizing particles to the overall energy-momentum balance of the system are not taken into account. Since the resulting inaccuracy of the energy and momentum conservation, as well as the intrinsic quantum-mechanical uncertainties of the energies and momenta of the involved particles, are completely negligible compared to their energies and momenta themselves, they can be safely ignored in most processes. This is, however, not justified when neutrino

⁴For example, in the case of Gaussian wave packets one finds $P_{\alpha\beta}(T, L) \propto \exp \left[- \frac{(L-vT)^2}{2\sigma_x^2} \right]$ [7, 4].

oscillations are considered, since in the absence of neutrino energy and momentum uncertainties the oscillations just would not occur. Therefore attempts to base the analyses of neutrino oscillations on exact energy-momentum conservation are inconsistent.

As we shall show now, exact energy-momentum conservation can play a role at an intermediate stage of QFT calculations of the transition amplitudes, when one deals with plane waves. But at this stage the oscillations cannot be obtained.

3.2 QFT and energy-momentum conservation

There are two approaches to the calculation of the amplitude of the neutrino production-propagation-detection process. In the first approach the external particles are described from the outset by localized wave packets in the configuration space, whereas in the second one the transition amplitude is first calculated in the plane-wave approach and then is convoluted with the momentum distribution functions of the external particles which characterize the momentum spreads of their wave packets and take into account their localization. As we shall show, the two approaches are actually equivalent; however, they allow different physics interpretations, in particular, regarding the role of energy-momentum conservation in neutrino oscillations. Let us consider these approaches in turn.

1. The external particles are described by localized configuration-space wave functions (wave packets). The transition amplitude corresponding to the production and detection of the i th neutrino mass eigenstate ν_i is given in (1). The quantity $M_i(x_1, x_2)$ in this equation is proportional to the propagator of ν_i . Due to invariance with respect to space-time translations, $M_i(x_1, x_2) = M_i(x_2 - x_1)$, and therefore it can be represented as a Fourier integral

$$M_i(x_2 - x_1) = \int \frac{d^4 q}{(2\pi)^4} M_i(q) e^{-iq(x_1 - x_2)}. \quad (4)$$

The wave functions $\Psi_j(x, \mathbf{X}_S, T_S)$ can also be represented as Fourier transforms of the corresponding momentum-space wave functions $f_j(\mathbf{p}, \bar{\mathbf{p}}_j)$:

$$\Psi_j(x, \mathbf{X}_S, T_S) = \int \frac{d\mathbf{p}}{(2\pi)^3} f_j(\mathbf{p}, \bar{\mathbf{p}}_j) e^{-i\varepsilon_j E_j(\mathbf{p})(t - T_S) + i\varepsilon_j \mathbf{p}(\mathbf{x} - \mathbf{X}_S)}. \quad (5)$$

Here $\varepsilon_j = +1$ stands for the initial-state particles and -1 for the final-state ones, and $f_j(\mathbf{p}, \bar{\mathbf{p}}_j)$ is the momentum distribution function of the j th external particle participating in neutrino production. It is characterized by a peak which is centered at $\mathbf{p} = \bar{\mathbf{p}}_j$ and has the width $\sigma_{pj} \sim \sigma_{xj}^{-1}$, where σ_{xj} is the spatial length of the wave packet Ψ_j . The latter follows from Heisenberg's uncertainty relation, and at the mathematical level is a reflection of the well-known property of the Fourier transformation.

The choice of the t - and \mathbf{x} -independent phase factor ($e^{i\varepsilon_j [E_j(\mathbf{p})T_S - i\mathbf{p}\mathbf{X}_S]}$) in the integrand of eq. (5) corresponds to the condition that the peak of the wave packet in the configuration space is at the point $\mathbf{x} = \mathbf{X}_S$ at the time $t = T_S$ (indeed, away from the point

$(t, \mathbf{x}) = (T_S, \mathbf{X}_S)$ the integrand becomes fast oscillating, leading to a strong suppression of the integral). This phase factor can be absorbed into a redefinition of the momentum distribution function, *i.e.* $f_j(\mathbf{p}, \bar{\mathbf{p}}_j) \rightarrow \tilde{f}_j(\mathbf{p}, \bar{\mathbf{p}}_j; T_S, \mathbf{X}_S)$:

$$\tilde{f}_j(\mathbf{p}, \bar{\mathbf{p}}_j; T_S, \mathbf{X}_S) = f_j(\mathbf{p}, \bar{\mathbf{p}}_j) e^{i\varepsilon_j [E_j(\mathbf{p})T_S - i\mathbf{p}\mathbf{X}_S]}. \quad (6)$$

When expressed through $\tilde{f}_j(\mathbf{p}, \bar{\mathbf{p}}_j; T_S, \mathbf{X}_S)$, the Fourier integral in (5) takes a more familiar form, with the phase factor $e^{-i\varepsilon_j [E_j(\mathbf{p})t - \mathbf{p}\mathbf{x}]} \equiv e^{-i\varepsilon_j p_j x}$ in the integrand. In addition to the information on the mean momentum and the width and shape of the peak, the momentum distribution function $\tilde{f}_j(\mathbf{p}, \bar{\mathbf{p}}_j; T_S, \mathbf{X}_S)$ encodes the information on the location of the center of the wave packet in the configuration space at a fixed time T_S .

Similar Fourier expansions and similar considerations apply also to the wave functions of the external particles participating in neutrino detection. Substituting these expansions as well as eqs. (5) and (6) into expression (1) for the transition amplitude, we find

$$\begin{aligned} \mathcal{A}_i = \int d^4x_1 d^4x_2 \prod_j \int \frac{d\mathbf{p}_j}{(2\pi)^3} \tilde{f}_j(\mathbf{p}_j, \bar{\mathbf{p}}_j; T_S, \mathbf{X}_S) \prod_l \int \frac{d\mathbf{p}_l}{(2\pi)^3} \tilde{f}_l(\mathbf{p}_l, \bar{\mathbf{p}}_l; T_D, \mathbf{X}_D) \\ \times M_i(x_2 - x_1) e^{-i\varepsilon_j p_j x_1 - i\varepsilon_l p_l x_2}. \end{aligned} \quad (7)$$

From this equation one can see that the mean values of the energies and momenta of the involved particles satisfy only *approximate conservation laws*. Indeed, if the momentum distribution functions $\tilde{f}_j(\mathbf{p}_j, \bar{\mathbf{p}}_j; T_S, \mathbf{X}_S)$ and $\tilde{f}_l(\mathbf{p}_l, \bar{\mathbf{p}}_l; T_D, \mathbf{X}_D)$ were proportional to, respectively, $\delta^{(3)}(\mathbf{p}_j - \bar{\mathbf{p}}_j)$ and $\delta^{(3)}(\mathbf{p}_l - \bar{\mathbf{p}}_l)$, the integrations over $d\mathbf{p}_j$ and $d\mathbf{p}_l$ would be removed by these δ -functions, and the remaining integral over $d^4x_1 d^4x_2$ of the expression in the second line of eq. (7) would result in the standard energy-momentum conserving $\delta^{(4)}$ -function. In reality, the momentum distribution functions are not of δ -type, but they are peaked at the corresponding mean momenta, with the peak widths $\sigma_p \ll |\bar{\mathbf{p}}|$. Therefore the mean energies and momenta of the particles involved in the neutrino production and detection processes satisfy only approximate conservation laws, with deviations from exact conservation determined by the values of the widths σ_p of the momentum distribution functions.

2. Alternatively, one can first calculate the transition amplitude in the plane-wave approach (*i.e.*, considering the external particles to be described by plane waves), and then convolute the obtained result with the actual momentum distribution functions of the external particles:

$$\mathcal{A}_i = \prod_j \int \frac{d\mathbf{p}_j}{(2\pi)^3} \tilde{f}_j(\mathbf{p}_j, \bar{\mathbf{p}}_j; T_S, \mathbf{X}_S) \prod_l \int \frac{d\mathbf{p}_l}{(2\pi)^3} \tilde{f}_l(\mathbf{p}_l, \bar{\mathbf{p}}_l; T_D, \mathbf{X}_D) \mathcal{A}_i^{pw}(\{p_j\}, \{p_l\}). \quad (8)$$

The plane-wave amplitude of the process, $\mathcal{A}_i^{pw}(\{p_j\}, \{p_l\})$, is given by

$$\mathcal{A}_i^{pw}(\{p_j\}, \{p_l\}) = \int d^4x_1 d^4x_2 M_i(x_2 - x_1) e^{-i(\sum_j \varepsilon_j p_j)x_1 - i(\sum_l \varepsilon_l p_l)x_2}. \quad (9)$$

This amplitude is proportional to a $\delta^{(4)}$ -function expressing the 4-momentum conservation in the process with external particles represented by plane waves. Indeed, substituting into (9) the Fourier expansion (4) for $M_i(x_2 - x_1)$ and performing the integration over q , we find

$$\mathcal{A}_i^{pw}(\{p_j\}, \{p_l\}) \propto \delta^{(4)}\left(\sum_j \varepsilon_j p_j + \sum_l \varepsilon_l p_l\right). \quad (10)$$

The amplitudes \mathcal{A}_i^{pw} , if substituted into eq. (2), would not lead to neutrino oscillations. This is related to the fact that plane waves are completely delocalized in space and time. The convolution in eq. (8) takes into account that the external particles are actually localized, and it is this localization that actually makes the oscillation possible.

It is easy to see that the described two approaches to the calculation of the transition amplitude are equivalent. Indeed, substituting (9) into (8) yields

$$\begin{aligned} \mathcal{A}_i = & \prod_j \int \frac{d\mathbf{p}_j}{(2\pi)^3} \tilde{f}_j(\mathbf{p}_j, \bar{\mathbf{p}}_j; T_S, \mathbf{X}_S) \prod_l \int \frac{d\mathbf{p}_l}{(2\pi)^3} \tilde{f}_l(\mathbf{p}_l, \bar{\mathbf{p}}_l; T_D, \mathbf{X}_D) \\ & \times \int d^4x_1 d^4x_2 M_i(x_2 - x_1) e^{-i\varepsilon_j p_j x_1 - i\varepsilon_l p_l x_2}. \end{aligned} \quad (11)$$

The expressions for the amplitudes in eqs. (7) and (11) coincide. Indeed, one of them is immediately obtained from the other by interchanging the order of the integrations over the momenta and 4-coordinates, $d^4x_1 d^4x_2$ and $\prod_{j,l} d\mathbf{p}_j d\mathbf{p}_l$.

The two expressions for the transition amplitude allow for different physical interpretations. The amplitude in eq. (7) is calculated with the coordinate-dependent localized wave functions of the external particles. From its construction it follows that the energies and momenta of the involved particles are not sharp, and their mean values satisfy only approximate conservation laws, as was discussed above. This is essential for neutrino oscillations to occur: indeed, as we have already discussed, too accurate measurements of neutrino energy and momentum would destroy neutrino oscillations.

On the other hand, the second approach emphasizes the relation of the calculation with the fact that energy and momentum conservations are actually exact laws of nature. It can be readily seen from (8) and (10) that the *individual* momentum components of the wave packets of the external particles (corresponding to the plane-wave amplitudes) satisfy exact energy and momentum conservation laws. The fact that the external particles are localized and conservation of their mean energies and momenta has only approximate nature is properly taken into account when one convolutes the plane-wave transition amplitude given by the second line of eq. (11) with the momentum distribution functions \tilde{f}_j and \tilde{f}_l , as is expressed by the outer integrals in (11). This is in line with the well known fact that, while plane waves are fully delocalized, integrating them over small intervals $\sim \sigma_p$ of momenta leads to constructive interference in certain space-time intervals of width $\sigma_x \sim 1/\sigma_p$ and destructive interference everywhere else.

4 Entanglement and the recoil phase

In refs. [8, 10, 11, 2, 12] it has been argued that exact energy-momentum conservation can still be applied to description of neutrino oscillations. This exact conservation leads to a kinematic entanglement of neutrinos with accompanying charged leptons. According to [10, 11, 2], the subsequent disentanglement is assumed to be due to the detection of the charged leptons or due to their interaction with the environment. This localizes the charged leptons and creates energy and momentum uncertainties for the neutrino state, which are necessary for the oscillations to occur. The disentanglement processes then has to be explicitly included in the description of neutrino oscillations. In what follows we will explore the relevance and physical meaning of this “entanglement/disentanglement” approach.

4.1 Kinematic entanglement and coherence

For definiteness we will speak about the $\pi \rightarrow \mu\nu$ decay, keeping in mind the general case of two-body decay $P \rightarrow R + \nu$, where P and R are the parent and recoil particles, respectively.

Suppose first that the 4-momentum of the pion p_π is well defined. Then, exact energy-momentum conservation would mean that, for each emitted neutrino mass eigenstate ν_i with a certain 4-momentum $p_{\nu i}$, the 4-momentum $p_{\mu i}$ of the accompanying muon must satisfy

$$p_{\nu i} + p_{\mu i} = p_\pi, \quad i = 1, 2 \quad (12)$$

(for simplicity we consider 2-flavor mixing here). In this way, the state produced in the pion decay can be a coherent superposition of different neutrino mass eigenstates accompanied by the muon states, 4-momenta of which are correlated with those of neutrinos (hence the name entangled state):

$$|\mu \nu\rangle = \sum_i U_{\mu i}^* |\mu(p_{\mu i})\rangle |\nu_i(p_{\nu i})\rangle. \quad (13)$$

If the 4-momentum of the muon is measured very accurately and the result gives, e.g., $p_{\mu 1}$, then the neutrino detector should observe only ν_1 with the 4-momentum $p_{\nu 1}$. This is a realization of the Einstein-Podolsky-Rosen (EPR) correlation [13]. In this case, however, no interference and therefore no oscillations would occur: Measuring the muon momentum with precision higher than $|\mathbf{p}_{\mu 1} - \mathbf{p}_{\mu 2}|$ would destroy the coherence of the components of the entangled state corresponding to different neutrino mass eigenstates.

As has been mentioned above, disentanglement amounts to a measurement of the muon momentum with a sufficiently large intrinsic uncertainty (due to a sufficiently good localization of the measurement process). Therefore, it implies a violation of the strict correlation (12) between the muon and neutrino 4-momenta, that is, it leads to a separation of the muon and neutrino parts of the state (13). On the other hand, coherence loss is a splitting of the components of the state (13) which correspond to different neutrino mass eigenstates.

We will show here that there actually exists an intimate relationship between the coherence and entanglement/disentanglement in the context of neutrino oscillations.

According to (2), the transition amplitude describing the oscillation process is the sum of the amplitudes which correspond to the emission and detection of ν_1 and ν_2 :

$$\mathcal{A}_{\alpha\beta}^{tot} = U_{\alpha 1}^* U_{\beta 1} \mathcal{A}_1(p_{\mu 1}, p_{\nu 1}) + U_{\alpha 2}^* U_{\beta 2} \mathcal{A}_2(p_{\mu 2}, p_{\nu 2}). \quad (14)$$

Consequently, the probability of the process, $|\mathcal{A}_{\alpha\beta}^{tot}|^2$, contains the interference term which is the real part of the expression

$$\begin{aligned} & U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2} \int_{D_\mu} d\mathbf{x}_\mu \mathcal{A}_1(p_{\mu 1}, p_{\nu 1})^* \mathcal{A}_2(p_{\mu 2}, p_{\nu 2}) \\ & \propto U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2} \int_{D_\mu} d\mathbf{x}_\mu e^{i(E_{\mu 1} - E_{\mu 2})t} e^{i(\mathbf{p}_{\mu 2} - \mathbf{p}_{\mu 1})\mathbf{x}_\mu}, \end{aligned} \quad (15)$$

which is responsible for the oscillations. Here the integration is over the muon detection region D_μ . If the size of the detector is smaller than $|\mathbf{p}_{\mu 1} - \mathbf{p}_{\mu 2}|^{-1}$, the integral is significantly different from zero and neutrino oscillations are observable; in the opposite case the integral is strongly suppressed due to the fast oscillations of the integrand, and the oscillations are averaged out.

In reality, the 4-momentum of the parent pion is never fixed precisely. The very fact that the pion decays implies that it is localized in space and time and hence its energy and momentum have uncertainties. Therefore, the pion must be described by a wave packet characterized by a momentum distribution function of a width $\sigma_{\pi p}$. This means that there is no strict correlation between the 4-momenta of the neutrino and muon produced in the pion decay. For a given value $p_{\nu i}$ of the 4-momentum of the i th neutrino mass eigenstate, the muon 4-momentum $p_{\mu i}$ is no longer uniquely determined by eq. (12); instead, it can take any value within a range of width of the order $\sigma_{\pi p}$. In other words, now instead of eq. (12) we have

$$p_{\nu i} + p_{\mu i} = p_{\pi i}, \quad i = 1, 2, \quad (16)$$

where $p_{\pi i}$ is no longer uniquely fixed. Consider, for example, the case when the 4-momenta of the components of the muons wave function accompanying the different neutrino mass eigenstates coincide, *i.e.* $p_{\mu 1} = p_{\mu 2} \equiv p_\mu$. This situation is realized when the muon is not detected, which corresponds to the limit $D_\mu \rightarrow \infty$, so that the expression in eq. (15) is proportional to $\delta^{(3)}(\mathbf{p}_{\mu 1} - \mathbf{p}_{\mu 2})$. One can then write

$$\mathbf{p}_\mu + \mathbf{p}_{\nu 1} = \mathbf{p}_{\pi 1}, \quad \mathbf{p}_\mu + \mathbf{p}_{\nu 2} = \mathbf{p}_{\pi 2}. \quad (17)$$

If

$$|\mathbf{p}_{\pi 1} - \mathbf{p}_{\pi 2}| \lesssim \sigma_{p\pi}, \quad (18)$$

the energy and momentum conservation laws are still satisfied for different components of the wave function, while no entanglement occurs. Indeed, under this condition one can

satisfy both equalities in eq. (17) simultaneously, which means that both neutrino mass eigenstates ν_1 and ν_2 can be produced with the muon having the same momentum \mathbf{p}_μ , and therefore the correlation between the 4-momenta of the neutrino and the muon is absent. At the same time, by making use of relations (17) we obtain from (18)

$$|\mathbf{p}_{\nu 1} - \mathbf{p}_{\nu 2}| \lesssim \sigma_{p\pi} . \quad (19)$$

This is the well known coherent production condition, which is a necessary condition for the observability of neutrino oscillations [14, 15, 4, 1]. It is essentially equivalent to the localization condition which requires that the pion decay region (*i.e.* the neutrino production region) be small compared to the oscillation length. Since the momenta of the two muon components may coincide, the integral in (15) is unsuppressed. There is no kinematic entanglement, the coherence condition is satisfied at production, and if the neutrino detection is also coherent, the oscillations can be observed. The observability of neutrino oscillations does not depend on the muon detection.

If $\sigma_{p\pi} \ll |\mathbf{p}_{\pi 1} - \mathbf{p}_{\pi 2}|$, the pion momentum is well defined and the neutrino and the muon are indeed produced in an entangled state. This reproduces approximately the extreme situation described in the beginning of this section (see eq. (12) and the following discussion)⁵. In this case the observability of neutrino oscillations will depend on the conditions of detection of the muon and, of course, of the neutrino. If the muon is not detected, the disentanglement does not occur. The amplitudes corresponding to different neutrino mass eigenstates do not interfere and neutrino oscillations do not occur, independently of the conditions of neutrino detection. What actually happens is that the oscillations are averaged out due to the integration over the coordinate of the pion decay point in the production region. The coherence condition is not satisfied at neutrino production and cannot be restored at detection. This can also be seen from eq. (15). Since the pion momentum is well defined and energy-momentum conservation is exact, the muon components have different momenta: $\mathbf{p}_{\mu 1} - \mathbf{p}_{\mu 2} \neq 0$. Therefore the infinite-volume integration in (15) leads to the vanishing result.

The situation is different when the muon is detected and therefore the muon momentum is measured with a finite resolution $\sigma_{p\mu}$. If

$$|\mathbf{p}_{\mu 1} - \mathbf{p}_{\mu 2}| < \sigma_{p\mu} , \quad (20)$$

the two muon components $\mu(p_{\mu 1})$ and $\mu(p_{\mu 2})$ interfere constructively in the detector, and the produced neutrino state can be a coherent superposition of different neutrino mass eigenstates. Condition (20) corresponds to the situation when the integration in (15) is performed over the domain of the linear size $|\Delta \mathbf{x}_\mu| \lesssim 1/|\Delta \mathbf{p}_\mu|$, so that the integrand does not exhibit fast oscillations and the integral is not suppressed. At the same time, condition

⁵ For pion decay and known neutrino mass squared differences this is not realized even for the decay of a free pion. So, here we consider a *gedanken* situation with either very large Δm^2 or a very long lived and strongly delocalized nucleus instead of pion.

(20) implies disentanglement of the neutrino and the muon. Indeed, both ν_1 and ν_2 can be produced together with the same muon state which has the momentum uncertainty $\sigma_{p\mu}$.

The observation of the muon in a finite-size detector provides its localization. This, together with the neutrino localization in its detection process, implies a localization of the decay region of the parent pion, *i.e.* of the neutrino production region. As a result, a washout of neutrino oscillations due to the averaging over the coordinate of the neutrino production point can be avoided. In other words, muon detection improves the localization of the pion decay region, thus improving the observability of neutrino oscillations.

If, however, $|\mathbf{p}_{\mu 1} - \mathbf{p}_{\mu 2}| \gg \sigma_{p\mu}$, the integral in (15) is essentially zero, *i.e.* the interference is absent independently of the conditions of neutrino observation. In this case the pion and muon momenta have well-defined values, and therefore energy-momentum conservation fully determines the 4-momentum of the emitted neutrino. Because the neutrino propagates macroscopic distances and consequently is on the mass shell, its energy and momentum in turn uniquely determine its mass. Thus, in this case only a certain mass-eigenstate (rather than flavor-eigenstate) neutrino would be emitted, which would make neutrino oscillations impossible.

The considerations presented in this section allow us to draw several general conclusions.

1. The absence of entanglement (or the occurrence of disentanglement) corresponds to the situation when the coherence condition at neutrino production is satisfied, and *vice versa*, entanglement (and the absence of subsequent disentanglement) implies violation of the coherence condition at neutrino production.

2. For neutrino oscillations to be observable, two different localization conditions must be satisfied: apart from the localization of the neutrino detection process, either the parent particle or the recoil particle at neutrino production should be localized (the latter, together with the localization of the neutrino detection, will automatically localize the production process as well).

3. In the case of disentanglement which restores the observability of neutrino oscillations the event should consist of muon interaction in a relatively small space-time region (apart from the neutrino detection). Just the fact that the muon interacts somewhere is not enough.

4. The kinematic disentanglement or the absence of entanglement occur when the energy and momentum uncertainties are introduced for the external particles which accompany neutrino production and detection. This is equivalent to describing the external particles by wave packets. Therefore, the treatment of neutrino oscillations in terms of possible entanglement and subsequent disentanglement can in a sense be considered as corresponding to the second QFT approach of sec. 3.2, where the transition amplitudes are first calculated using plane waves (which implies exact energy-momentum conservation) and then are convoluted with momentum-space wave packets. If the coordinate-space wave packets are used from the very beginning, kinematic entanglement is completely irrelevant. The attempts to use the entanglement/disentanglement approach within the pure plane-wave framework are inconsistent.

5. The entanglement/disentanglement approach can describe certain, but not all, situations. It does not describe the most important case when the parent pion has a large enough momentum uncertainty, so that the entanglement does not arise at all. In this case neutrino oscillations can be observed even if the muon is not detected (disentangled). Neutrino oscillations can then be described in a consistent way without resorting to kinematic entanglement and in this sense the entanglement/disentanglement approach is irrelevant.

6. The idea of making use of entanglement and subsequent disentanglement of the charged leptons and neutrinos stems from the desire to base the theory of neutrino oscillations on exact energy-momentum conservation, which is a rigorous law of nature. Let us stress once again, however, that the approximate conservation of mean energies and momenta in the wave packet approach does not contradict this exact conservation law. The approximate nature of conservation of the mean 4-momenta is related to the fact that the contributions to the overall energy and momentum balance coming from the surrounding particles, which localize the particles directly involved in the neutrino production and detection processes, are not taken into account. The directly involved particles form an open system. The energy and momentum conservation laws are, however, fulfilled exactly for contributions to the transition amplitudes coming from the individual momentum components of the wave packets of these particles [4] (sec. 3).

4.2 The phases of the recoil particles

In a number of papers [12, 16, 17] it has been argued that kinematic entanglement implies that there are certain contributions of the phases ϕ_R of the recoil particles accompanying the neutrino production to the oscillation phase. (In [2] it is assumed that the phase ϕ_R is small since the recoil is detected near the source.) Let us clarify this issue. Consider again the pion decay. For simplicity, we will consider only one spatial dimension, so that

$$p_\alpha = \{E_\alpha, k_\alpha\}, \quad x_\alpha = \{t_\alpha, r_\alpha\} \quad (\alpha = \pi, \mu, \nu),$$

keeping the possibility of the \pm signs for momenta and coordinates (the generalization to the 3-dimensional case is straightforward). Suppose that the pion decays at the point $r = 0$ at the time $t = 0$, and that the muon and neutrino are detected at the points with 4-coordinates $x_\mu = (t_\mu, r_\mu)$ and $x_\nu = (t_\nu, r_\nu)$, respectively. To the time of detection the state (13) evolves into

$$|\mu(x_\mu)\nu(x_\nu)\rangle = \sum_i U_{\mu i}^* |\mu(p_{\mu i})\rangle |\nu_i(p_{\nu i})\rangle e^{i(\phi_{\nu i}(x_\nu) + \phi_{\mu i}(x_\mu))}, \quad (21)$$

where

$$\phi_{\nu i}(x_\nu) = k_{\nu i} r_\nu - E_{\nu i} t_\nu, \quad \phi_{\mu i}(x_\mu) = k_{\mu i} r_\mu - E_{\mu i} t_\mu. \quad (22)$$

The total phase difference between the two components of the state (21) corresponding to different neutrino mass eigenstates is

$$\Delta\phi = \Delta\phi_\nu(x_\nu) + \Delta\phi_\mu(x_\mu), \quad (23)$$

where

$$\Delta\phi_\nu \equiv \phi_{\nu 2} - \phi_{\nu 1}, \quad \Delta\phi_\mu \equiv \phi_{\mu 2} - \phi_{\mu 1}. \quad (24)$$

According to (23) the probability of neutrino oscillations should depend on the 4-coordinate of the point where the accompanying muon is detected. However, in all realistic setups the phase difference $\Delta\phi_\mu(x_\mu)$ between the components of the muon state *is not measured*. Even if $\Delta\phi_\mu(x_\mu)$ were measured, this would not influence the neutrino oscillation pattern. Indeed, by itself the fact of the muon detection (irrespectively of whether or not the phase difference between its different components is measured) only establishes that *the neutrino produced in the source was the muon neutrino*. This fixes the composition of this initial neutrino state in terms of the mass eigenstates. The subsequent change of the relative phase between the mass eigenstates (which leads to neutrino oscillations) is entirely determined by $\Delta m^2/2E_\nu$ and by the distance traveled by the neutrinos.

Thus, we have a paradoxical situation: formally, the phase of the recoil particle enters into the expression for the entangled state. At the same time, the pattern of neutrino oscillations should not depend on this phase. As we will see, the resolution of this paradox is trivial: *The recoil phase is negligible under the conditions of observability of neutrino oscillations*. This phase is a part of the uncertainty of the oscillation phase due to the finite extension of the neutrino source.

Let us discuss the muon phase difference in more detail. In general,

$$\Delta\phi_\mu(t_\mu, r_\mu) = \Delta k_\mu r_\mu - \Delta E_\mu t_\mu, \quad (25)$$

where Δk_μ and ΔE_μ are the momentum and energy differences of the muon components which correspond to two different neutrino mass eigenstates. From the mass shell relation $k_\mu^2 = E_\mu^2 - m_\mu^2$ we obtain

$$\Delta k_\mu = \Delta E_\mu \frac{E_\mu}{k_\mu}. \quad (26)$$

Substituting this into (25) yields

$$\Delta\phi_\mu(x_\mu) = \Delta E_\mu \left(\frac{E_\mu}{k_\mu} r_\mu - t_\mu \right) = \frac{\Delta E_\mu}{v_\mu} (r_\mu - v_\mu t_\mu), \quad (27)$$

where $v_\mu = k_\mu/E_\mu$ is the muon velocity. If one assumes the muon to be a pointlike particle moving along a classical trajectory, the two terms in the brackets in eq. (27) cancel each other, so that the contribution of the recoil to the phase difference (23) vanishes: $\Delta\phi_\mu = 0$. Let us stress, however, that the fact that we assume all particles to have well-defined momenta implies that they are described by plane waves – a notion which is just the opposite of pointlike particles. Thus, the argument presented above is actually controversial. Plane waves also bear other well known difficulties – in particular, if the pion energy and momentum are well defined, the pion is completely delocalized in space and time, so that the distances from the muon and neutrino production to detection points x_μ and x_ν cannot be defined.

In a consistent wave packet formalism, the equality $r_\mu = v_\mu t_\mu$ is satisfied (and the recoil phase vanishes) only at the center of the muon wave packet. In general, $r_\mu \neq v_\mu t_\mu$, but $|r_\mu - v_\mu t_\mu|$ cannot exceed significantly the spatial length of the muon wave packet $\sigma_{x\mu}$. Consequently,

$$|\Delta\phi_\mu| = \frac{\Delta E_\mu}{v_\mu} |r_\mu - v_\mu t_\mu| \lesssim \frac{\Delta E_\mu}{v_\mu} \sigma_{x\mu}, \quad (28)$$

which is valid for all r_μ or t_μ . The spatial length of the muon wave packet is related to its energy uncertainty as $\sigma_{x\mu} \sim 1/\sigma_{k\mu} = v_\mu/\sigma_{E\mu}$. Then, eq. (28) can be rewritten as

$$|\Delta\phi_\mu| \lesssim \frac{\Delta E_\mu}{\sigma_{E\mu}}. \quad (29)$$

This expression means that if the coherence condition $\Delta E_\mu \ll \sigma_{E\mu}$, which is required for observability of neutrino oscillations, is fulfilled, the phase $\Delta\phi_\mu$ is negligible. (Recall that according to the coherence condition the energy and momentum uncertainties of the particles accompanying neutrino production and detection should be large enough, so that the momenta of different neutrino mass eigenstates can not be distinguished).

For neutrinos, instead of eq. (26) we have

$$\Delta k_\nu = \Delta E_\nu \frac{E_\nu}{k_\nu} - \frac{\Delta m^2}{2k_\nu}, \quad (30)$$

and consequently the neutrino phase difference is

$$\Delta\phi_\nu(x_\mu) = \frac{\Delta E_\nu}{v_\nu} (r_\nu - v_\nu t_\nu) - \frac{\Delta m^2}{2k_\nu} r_\nu. \quad (31)$$

The key difference between $\Delta\phi_\mu$ and $\Delta\phi_\nu$ originates from the fact that both components of the muon state have the same mass, whereas the components of the neutrino state have different masses and therefore an additional contribution to $\Delta\phi_\nu$ appears due to the neutrino mass difference Δm^2 (cf. eqs. (27) and (31)). The first term in (31) can be estimated just as in the case of the muon wave packet: it vanishes at the center of the neutrino wave packet, is non-zero away from this point, but never exceeds significantly the spatial length of the neutrino wave packet $\sigma_{x\nu} \sim v_\nu/\sigma_{E\nu}$, where $\sigma_{E\nu}$ is the energy uncertainty characterizing the neutrino state. Therefore

$$\frac{\Delta E_\nu}{v_\nu} |r_\nu - v_\nu t_\nu| \lesssim \frac{\Delta E_\nu}{\sigma_{E\nu}}. \quad (32)$$

Combining eqs. (23), (27) and (31) we obtain for the total phase difference

$$\Delta\phi = \frac{\Delta E_\mu}{v_\mu} (r_\mu - v_\mu t_\mu) + \frac{\Delta E_\nu}{v_\nu} (r_\nu - v_\nu t_\nu) - \frac{\Delta m^2}{2k_\nu} r_\nu. \quad (33)$$

From eqs. (28) and (32) it follows that the first two terms in (33) are negligible when the energy differences of the different components of the muon and neutrino states are

small compared to their respective energy uncertainties, *i.e.* when the coherent neutrino production condition is satisfied, as advertised.

In a complete QFT approach the phase of parent particle, that is, the pion phase $\Delta\phi_\pi(x_\pi)$ in our example, also should be taken into account. Now instead of (23), we have ⁶

$$\Delta\phi = -\Delta\phi_\pi(x_\pi) + \Delta\phi_\nu(x_\nu) + \Delta\phi_\mu(x_\mu), \quad (34)$$

where the minus sign in front of $\Delta\phi_\pi$ is related to the fact that the pion is destroyed, whereas the muon and the neutrino are produced. Here $x_\pi = (t_\pi, r_\pi)$, and all distances and times are counted from the point of the pion decay. The expression for $\Delta\phi_\pi$ is similar to that for the muon, and therefore for the total phase difference we obtain

$$\Delta\phi = -\frac{1}{v_\pi}(r_\pi - v_\pi t_\pi)\Delta E_\pi + \frac{1}{v_\mu}(r_\mu - v_\mu t_\mu)\Delta E_\mu + \frac{1}{v_\nu}(r_\nu - v_\nu t_\nu)\Delta E_\nu - \frac{\Delta m^2}{2k_\nu}r_\nu. \quad (35)$$

Equation (35) is equivalent to eq. (39) of Dolgov *et al.* [9], where the dependence of the oscillation phase on the 4-coordinate of the muon was interpreted as an indication of the EPR-type correlation in neutrino oscillations in the case when both muon and neutrino are detected. While we agree with that equation, we disagree with its interpretation given in [9].

Indeed, the pion phase difference can be estimated similarly to that for the muon. As a result, we find that each of the first three terms on the right hand side of eq. (35) is smaller than or of the order of $\Delta E_\alpha/\sigma_{E\alpha}$ ($\alpha = \pi, \mu, \nu$). The condition of coherent neutrino production requires that all the involved energy differences be small compared to the corresponding energy uncertainties (otherwise one would have been able to determine which neutrino mass eigenstate was emitted). Hence, if the coherent production condition is satisfied, the first three terms on the right hand side of eq. (35) are small compared to unity and can be neglected; the remaining (last) term just gives the standard oscillation phase. We therefore conclude that the condition of coherent production eliminates the dependence of the oscillation phase on the 4-coordinate of the muon, and no EPR-like correlations arise. On the other hand, if the coherence condition is violated, neutrino oscillations are unobservable.

The effect of the recoil particle can be understood from fig. 2. If the muon is not detected or the duration of the detection process exceeds τ_π , the oscillation diagram coincides with the one shown in the figure. In this case the oscillation pattern is not affected by the recoil particle: it is determined by the localization of the pion and by the neutrino parameters. If the muon detection proceeds during a shorter time interval, $t_{det} < \tau_\pi$, the muon (green) band will be narrower. Correspondingly, the production (overlap) region will be shorter. That, in turn, will reduce the width of the neutrino wave packet and consequently will improve the production coherence.

⁶Equation (34) can be obtained by considering the amplitudes in eq. (8).

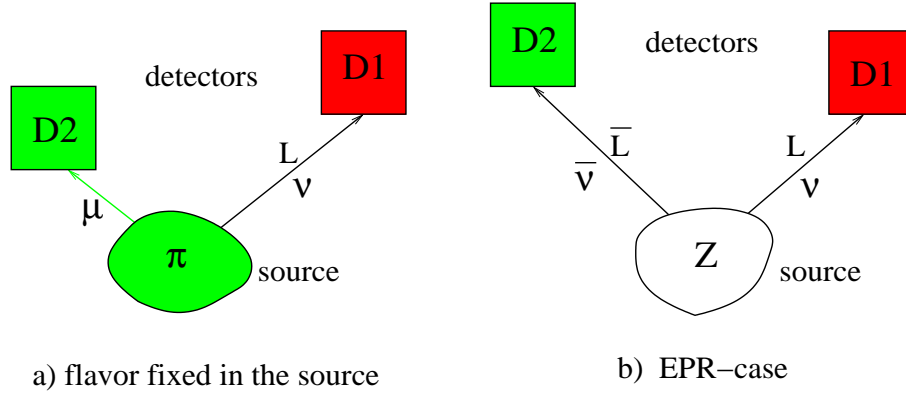


Figure 3: Two-detector neutrino oscillation experiments: Neutrino production along with μ in a pion decay (a) and $\nu\bar{\nu}$ production in a Z^0 decay (b). EPR correlation is present only in the second case.

4.3 Recoil particle phase and the EPR paradox

Let us compare the situation described in the previous subsection with the case of neutrinos produced in Z^0 decays. Here the conditions for the EPR paradox are realized and the phases of both products of the decay are important for the oscillations [18]. The neutrino state produced in the Z^0 decay is given by

$$|\nu_Z\rangle = \frac{1}{\sqrt{3}} \sum_i |\bar{\nu}_i \nu_i\rangle. \quad (36)$$

The neutrino mass is not determined; all mass eigenstates are produced with the same probability amplitude. The mass eigenstates are entangled. Let us consider a *gedanken* experiment with two neutrino detectors sensitive to the neutrino mass. If the first detector observes a mass-eigenstate component ν_i , then the second detector should see the antineutrino $\bar{\nu}_i$ of the same mass, independently of the distance between the detectors and source.

The situation with the flavor neutrino states is more complicated, since these states are not eigenstates of the free Hamiltonian, *i.e.* they may oscillate. The Z^0 -boson interactions are flavor blind and therefore all three flavors are produced with the same probability amplitude. Indeed, from $|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$ it follows that (36) can be rewritten as

$$|\nu_Z\rangle = \frac{1}{\sqrt{3}} \sum_{\alpha=e,\mu,\tau} |\bar{\nu}_\alpha \nu_\alpha\rangle \quad (37)$$

and is independent of the elements of the leptonic mixing matrix $U_{\alpha i}$. The flavor states correlate at the production: if both detectors are located near the production region and, *e.g.*, ν_e is detected by the first detector, then in the second detector $\bar{\nu}_e$ should be observed. If the detectors is situated at large enough distance from the source, so that neutrinos have

time to oscillate, the flavor correlations will depend on the distance between the source and the detectors (see fig. 3). If one of the detectors observes, e.g., ν_μ , this will fix the flavor composition of the antineutrino state observed by the second detector. It will be determined by the sum of the phase differences acquired by neutrinos and antineutrinos on their ways from the source to the detectors [18]:

$$\Delta\phi = \Delta\phi_\nu + \Delta\bar{\phi}_\nu. \quad (38)$$

The oscillation picture corresponds to the situation as if one of the detectors was a source of the neutrino and the other one a detector.

The key condition for the observability of neutrino oscillations is that the flavor composition of the neutrino state be measured in two different space-time points (fig. 3). If the neutrino flavor is fixed in one space-time point, then the oscillatory pattern can be observed in another space-time point. In the case under consideration, the source (Z^0 decay) does not fix the neutrino flavor, and therefore two flavor detectors are necessary. The difference of this EPR setup from neutrino production in, e.g., $\pi \rightarrow \mu\nu$ decay is that in the latter case the flavor of the neutrino state is fixed at production independently of the place where the muon is detected, and even of whether the muon is actually detected or not.

5 Conclusions

In this paper we have considered the related issues of entanglement, energy-momentum conservation and possibility of consistent description of neutrino oscillations in the S -matrix formalism.

1. We have shown that neutrino oscillations, being intrinsically a finite space-time phenomenon, can be consistently described in the S -matrix formalism provided that the correct physics conditions of the oscillation setup are imposed. The key condition is the existence of two finite space-time interaction regions: the production region and the detection region. The S -matrix formalism should be adjusted correspondingly. In the context of QFT this adjustment requires using wave packets for the external particles, i.e. the particles accompanying neutrino production and detection. This allows one to describe their localization, and consequently, define the neutrino production and detection regions. The wave packets encode information about the interactions of the particles accompanying neutrino production and detection with the external (to the neutrino processes) system which provides the localization of these particles.

Instead of using wave packets for the external particles, one can employ plane waves amended by finite-interval space-time integrations over the 4-coordinates of the neutrino production and detection points. This can be considered as an approximation to the description of neutrino oscillations in a realistic wave packet framework. Since finite pieces of plane waves are in fact wave packets, this approach can actually be considered as a primitive wave packet one.

2. We have demonstrated the equivalence of two approaches to calculating the oscillation amplitude: (i) Using configuration-space wave packets for the external particles involved in neutrino production and detection. (ii) Using plane waves for the external particles and the subsequent convolution of the obtained amplitudes with the wave packets of the external particles in the momentum representation. These two approaches allow different physics interpretation. The first one underlines the approximate nature of conservation of the mean energies and momenta of the involved particles. In the second approach one can speak of exact energy-momentum conservation for the individual momentum components of the wave packets. However, these individual components appear only at an intermediate stage of calculations, and at this stage the oscillations cannot be obtained. The required localization of the neutrino production and detection processes and, consequently, the oscillations are only obtained after the convolution of the plane-wave amplitudes with the momentum-space wave packets of the external particles which encode the information about their localization.

3. The assumption that particles involved in neutrino production processes have well-defined energies and momenta which satisfy exact conservation laws leads to kinematic entanglement of the neutrinos and the accompanying particles (e.g., charged daughter particles in decays). This entanglement destroys coherence of a neutrino state and therefore oscillations become unobservable unless energy and momentum uncertainties are introduced for the recoil and/or parent particles. To observe the oscillations, one should arrange localization in two different sites of the experimental setup: at neutrino production (the parent and/or recoil particles should be localized) and at detection (the neutrino absorption process should be localized). This provides the requisite energy and momentum uncertainties which make neutrino oscillations possible.

The conditions for entanglement and disentanglement are related to the coherence condition at neutrino production and detection. In the case when the production process is delocalized, energy-momentum conservation leads to kinematic entanglement of neutrinos and recoil particles but prevents coherent emission of different neutrino mass eigenstates and therefore makes the oscillations unobservable. Disentanglement of neutrinos and recoil particles through the detection of the latter can restore the coherence of the emitted neutrino state. If, however, the localization of the parent particle is good enough, so that the coherence condition at the neutrino production is satisfied, no entanglement occurs, and the corresponding description is irrelevant. A complete and consistent description of the oscillations is then possible without resorting to the entanglement/disentanglement approach.

4. We show that, for processes induced by charged-current weak interactions, the phases of the recoil particles (e.g., of charged leptons associated with neutrino production or detection) are irrelevant for neutrino oscillations. If the coherent production condition is fulfilled, the contribution of the recoil particle's phase to the oscillation phase is negligible and, in fact, is included in the uncertainty related to the finite extension of the source. No EPR-like effects arise. This differs from the case of neutrino production by neutral currents (e.g. in Z^0 decay). Here the flavor of the neutrino state is not fixed at the production point, and the oscillations can only be observed in a two detectors experiment, when one of the

detectors fixes the flavor composition of the neutrino state and the other one observes the oscillation pattern in the antineutrino state (or *vice versa*). In contrast to this, in charged current processes the neutrino flavor is already fixed at production, and therefore neutrino oscillations do not depend on the evolution of the recoil particles.

Appendix A: Finite observation time approach of [2, 3]

In all the situations considered in [2] the physics setups did not correspond to those of realistic neutrino oscillation experiments. The authors used the plane wave formalism but did not introduce the correct limits of integration over the 4-coordinates of the neutrino emission and absorption points, which is mandatory in that case. Three different computations of the oscillation probabilities were presented in [2]:

1. In the case called “the S -matrix result”, the authors consider a single region of neutrino interactions and perform integrations over the 4-coordinates of the production and detection vertices in the same infinite intervals. This corresponds to a scattering rather than oscillation setup (fig. 1). Such an integration leads to averaging over all possible baselines L from 0 to ∞ , and consequently, the oscillations are averaged out.

2. In the second case the authors again took a single interaction region and used infinite spatial integration, but performed time integration over a finite interval from t^i to t^f . The time ordering has been imposed, so that the charged lepton associated with neutrino production is observed near the production point earlier than the charged lepton associated with neutrino detection which is observed near the neutrino absorption point. The integrations over the production and detection times t_1 and t_2 are performed over the same interval (t^i, t^f) with $t_1 \leq t_2$. This would correspond to the averaging (integration) over the baselines L from 0 to $v(t^f - t^i)$, where v is the neutrino velocity. Now, due to the finite time integration, the oscillatory terms appear in the expression for the transition probability, *i.e.* the oscillations are not averaged out. Thus, the results obtained in this case differ substantially from those in the previous case. From this the authors conclude that the S -matrix formalism is ill-suited to describe neutrino oscillations. However, both results are incorrect in the sense that they do not correspond to a realistic oscillations setup. As is discussed in sec. 2 of the present paper, with the correct setup the S -matrix formalism is perfectly adequate for describing neutrino oscillations.

3. In the third computation the authors assumed disentanglement of the neutrino and charged lepton in the final state of the production process and introduced two different intervals of time integrations. The integration over t_1 is performed from $t = 0$ (the moment of neutrino production) to t_S – the time of detection (disentanglement) of the accompanying charged lepton. This detection essentially occurs in the neutrino source, so that $vt_S \ll l_\nu$, where l_ν is the neutrino oscillation length. The time t_S is of the order of or larger than the production interval $t_f^S - t_i^S$ considered in our paper (see sec. 2). The second integration is from t_S^f to t_D^f (in our notation). This interval is simply attached to the production interval,

which is definitely incorrect for the long-baseline oscillation setup, provided that one uses plane waves. This integration corresponds to the situation when the wave functions of the external particles participating in neutrino detection are not localized inside the detector but are distributed uniformly all the way from the neutrino production region ($t \sim t_S^f$) to the detection region ($t \sim t_D^f$). That is, here one deals with a detector that is as long as the baseline itself. It is this time integration over the interval $t_S^f - t_D^f$ that leads in [2] to the oscillation probability which is actually the integral of the standard neutrino oscillation probability.

Appendix B: Entanglement and the oscillation length

In refs, [12, 17] two-body decays $P \rightarrow \nu + R$, were considered, where P is the parent particle and R is the “recoil”. The total phase difference between the ν_1 - and ν_2 -containing components of the entangled state $|\nu R\rangle$ is

$$\Delta\phi = \Delta\phi_\nu + \Delta\phi_R. \quad (\text{B1})$$

In turn, the phase differences $\Delta\phi_\nu$ and $\Delta\phi_R$ each have two contributions: the contribution from the difference of energies and the contribution from the difference of momenta, so that

$$\Delta\phi = (\Delta k_R r_R - \Delta E_R t_R) + (\Delta k_\nu r_\nu - \Delta E_\nu t_\nu), \quad (\text{B2})$$

where

$$\begin{aligned} \Delta E_R &\equiv E_{R2} - E_{R1}, & \Delta E_\nu &\equiv E_{\nu 2} - E_{\nu 1}, \\ \Delta k_R &\equiv k_{R2} - k_{R1}, & \Delta k_\nu &\equiv k_{\nu 2} - k_{\nu 1}. \end{aligned} \quad (\text{B3})$$

In sec. 4.2 we have shown that

$$|\Delta k_R r_R - \Delta E_R t_R| \lesssim \frac{\Delta E_R}{\sigma_{E_R}}, \quad (\text{B4})$$

where σ_{E_R} is the energy uncertainty of the state of the recoil particle. As shown in sec. 4.2, the coherent neutrino production condition, which is a necessary condition for the observability of neutrino oscillations, requires, in particular, $\Delta E_R/\sigma_{E_R} \ll 1$. Therefore eq. (B4) means that under the coherence condition one has $\Delta\phi_R \ll 1$, *i.e.* the term in the first brackets in (B2) can be neglected. Hence the total phase difference is simply reduced to the phase difference of the two neutrino mass eigenstates, $\Delta\phi \approx \Delta\phi_\nu$ which, in turn, leads to the standard oscillation phase and length. This result does not depend on the distance and time of detection of the recoil particle and it does not depend on whether or not the energy-momentum conservation is applied. Note that in ref. [17] the conclusion that the oscillation phase takes its standard value was obtained assuming that the neutrino and the

recoil particles move along classical trajectories, i.e. are in fact pointlike. This is not a consistent assumption because the consideration in [12, 17] was based on the plane wave approach, and the notion of pointlike particles is just the opposite of that of plane waves.

Most of the complications encountered in refs. [12, 17] originate essentially from the following grouping of the terms in the expression for the total phase difference (B2):

$$\Delta\phi = (\Delta k_R r_R + \Delta k_\nu r_\nu) - (\Delta E_R t_R + \Delta E_\nu t_\nu), \quad (\text{B5})$$

which differs from that in (B2). Following [12], we shall consider the problem in the rest frame of the parent particle P (the generalization to the case of P decay in flight is trivial). In [12] exact energy-momentum conservation was applied and it was assumed that

$$t_R = t_\nu \equiv t, \quad (\text{B6})$$

i.e. that the moments of time at which the recoil and the neutrino are observed coincide in the rest frame of P . In principle, this is possible, in general this is not justified, and in practice this is never realized. Under condition (B6), for the expression in the second brackets of eq. (B5) one obtains

$$\Delta E_R t_R + \Delta E_\nu t_\nu = (\Delta E_R + \Delta E_\nu) t = 0. \quad (\text{B7})$$

The second equality here holds because, due to energy conservation, one has $E_{R2} + E_{\nu 2} = E_{R1} + E_{\nu 1} = m_P$, where m_P is the mass of the parent particle, so that $\Delta E_R = -\Delta E_\nu$. Consequently,

$$\Delta\phi(t) = \Delta k_R r_R + \Delta k_\nu r_\nu = \Delta k_\nu (r_\nu - r_R), \quad (\text{B8})$$

where it has been taken into account that due to the momentum conservation $k_{Ri} = -k_{\nu i}$ ($i = 1, 2$) and therefore, $\Delta k_R = -\Delta k_\nu$. Then in [12] it was concluded that according to eq. (B8) one should observe the oscillation pattern with the oscillation length

$$l_{R\nu} = \frac{2\pi}{\Delta k_\nu}. \quad (\text{B9})$$

Here Δk_ν is determined by the kinematics of the decay and depends on the masses of the particles involved. Consequently, $l_{R\nu}$ differs from the standard oscillation length and turns out to be process-dependent.

Subsequently, in ref. [16] the length $l_{R\nu}$ was called the “separation wavelength” which determines the oscillatory pattern with respect to the separation $D \equiv r_\nu - r_R$ between the neutrino and the recoil. Yet another oscillation length has been introduced – “the oscillation length relative to the decay point”. The phase (B8) can be rewritten in terms of the distance traveled by the neutrino in the standard form

$$\Delta\phi(t) = \Delta k_\nu \left(1 - \frac{r_R}{r_\nu}\right) r_\nu = 2\pi \frac{r_\nu}{l_\nu}, \quad (\text{B10})$$

where

$$l_\nu \equiv l_{R\nu} \frac{1}{1 - \frac{r_R}{r_\nu}}. \quad (\text{B11})$$

Since r_R and r_ν are measured in the same moment of time t in the rest frame of the parent particle, for pointlike neutrinos and recoil particles these two distances are related, and their ratio is determined by the velocities of the involved particles. It is then straightforward to show that l_ν coincides with the standard oscillation length.

Our point is that all these complications and the introduction of the “separation” oscillation length have no physical meaning. If the coherent production condition is satisfied, the additional “recoil” term $\Delta k_\nu r_R$ in the expression for $\Delta\phi(t)$ (see (B8)) is essentially nothing but the contribution to the oscillation phase coming from the difference of neutrino energies: $\Delta k_\nu r_R \approx \Delta E_\nu t$. Indeed, we have

$$\Delta k_\nu r_R = -\Delta k_R r_R \approx -\Delta E_R t = \Delta E_\nu t, \quad (\text{B12})$$

where the first equality follows from the momentum conservation, the second (approximate) equality – from eq. (B4) and the coherence condition $\Delta E_R/\sigma_{E_R} \ll 1$ (which imply $\Delta k_R r_R \approx \Delta E_R t$), and the last one is due to energy conservation. As a result, the expression for the oscillation phase takes the usual form

$$\Delta\phi = \Delta k_\nu r_\nu - \Delta E_\nu t_\nu \quad (\text{B13})$$

with only the neutrino contribution present.

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